

Fibonacci is All Around

I. UNIT OVERVIEW & PURPOSE:

The overall purpose of this activity is to explore the many wonders of the Fibonacci Sequence and see how the sequence is related to the Golden Ratio in our own natural habitat. The main focus group is for Algebra 1 or Geometry students to build a better understanding of finding patterns and relationships between patterns and how they can be used with real-world application.

II. UNIT AUTHOR:

Lynn Miller-Jones, Staunton River Middle School, Bedford County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Algebra, Geometry

V. OBJECTIVES:

Students will explore and investigate how to generate the Fibonacci sequence and discover how its unique attributes produce the Golden Ratio. Students will then use the Golden ratio created from the Fibonacci Sequence to identify how it appears in nature. Finally students will explore the use of a Fibonacci Gauge to help create “golden” materials.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE. 1 The student will solve practical problems involving rational numbers (including numbers in scientific notation), percent, ratios, and proportions.

MPE. 3. The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation.

MPE. 7. The student will use similar geometric objects in two- or three-dimensions to solve real-world problems about similar geometric objects.

MPE. 10. The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n th term, and evaluating summation formulas.

VII. CONTENT:

- Lesson 1 will involve having the students explore the Fibonacci Sequence and then using an excel file to generate the Golden Ratio.
- Lesson 2 will involve the students gathering information regarding how the Golden Ratio appears in nature.
- Lesson 3 will involve students creating a Fibonacci Gauge and using it to identify items within the room that meet the Golden Ration and then using the gauge to create a drawing.

VIII. REFERENCE/RESOURCE MATERIALS:

Students will need to have access to a computer for internet research and excel computations; rulers and grid paper; and various nature objects such as pine cones and flowers.

IX. PRIMARY ASSESSMENT STRATEGIES:

Students will be assessed based on research data, computation of individual works, observation of work habits and overall project completion.

X. EVALUATION CRITERIA:

Grading rubric is included with each lesson.

XI. INSTRUCTIONAL TIME:

Lesson 1: One 90 minute class period

Lesson 2: One 90 minute class period

Lesson 3: One 90 minute class period

Exploring the Fibonacci Sequence

Strand Algebra, and Geometry

Mathematical Objective

Students will explore and investigate how to generate the Fibonacci sequence and discover how its unique attributes produce the Golden Ratio.

Mathematics Performance Expectation(s)

MPE. 1 The student will solve practical problems involving rational numbers (including numbers in scientific notation), percent, ratios, and proportions.

MPE. 10. The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n th term, and evaluating summation formulas.

Related SOL

- A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.
- G.14 The student will use similar geometric objects in two- or three-dimensions to compare ratios between side lengths, perimeters, areas, and volumes and solve real-world problems about similar geometric objects.

NCTM Standards

- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- relate and compare different forms of representation for a relationship;
- solve problems involving scale factors, using ratio and proportion;
- use geometric models to represent and explain numerical and algebraic relationships;
- recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life;
- Recognize and apply mathematics in contexts outside of mathematics.

Materials/Resources (per group)

- Domino style sets of ten tiles for each pair of students
- Grid paper
- Access to an excel program
- *Exploring Fibonacci* worksheet

Assumption of Prior Knowledge

- Student must have an understanding of how to create an array for multiplication.
- Students must be able to analyze a pattern and produce a rule.
- Students need to have an understanding of how to create formulas in an Excel program.

Introduction: Setting Up the Mathematical Task

This activity is designed for students to explore the Fibonacci Sequence and make a conjecture about what ratio the sequence produces.

Duration: This project will take approximately one 90 minute class.

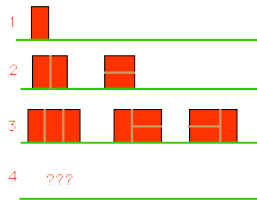
Student Exploration 1:**Introduction: (10 minutes)**

To introduce the activity, have students explore the beginning of the sequence for the existence of a pattern: 1, 1, 2, 3, 5, 8, 13 ... and then extend the pattern to the next 5 numbers in the sequence. Discuss the findings of the students and have them explain how they got the remaining numbers in the sequence.

Small Group Work (30 minutes)

1. Place students into pairs and provide a set of ten domino style rectangles. Distribute the "Exploring Fibonacci" worksheet
2. Explain to the students they will be exploring how many possibilities there are to arrange a rectangle that measures 2 x 10. Make sure to inform students that the positioning of the bricks (tiles) makes a difference.

3. The students will begin by working with a 2 x 1 rectangle and work their way up to a 2 x 10. Have the students explore the existence of a pattern in the chart.



4. Challenge the students to find Fibonacci sequence in the following examples:
- Pascal's Triangle
 - One octave level in a set of piano keys.
 - Set of branches on a tree

Whole Class Sharing/Discussion

Discuss findings of students. Possibly have students display their grid arrangements under a document camera. Have students explain where they see the sequences in each of the problems above.

Student Exploration 2:

Individual Computer Work (30 – 40 minutes)

1. Using an excel program, have the students generate the first 20 numbers of the Fibonacci Sequence using a rule. (For the purpose of the excel file, have the students generate the rule using the 2nd and 3rd terms in the sequence.)
 - a. Column A will be used to identify the index number in the sequence
 - b. Column B will be the Fibonacci Sequence
2. Have the students create a third column that creates the ratio of $\frac{\text{next term in the sequence}}{\text{current term in the sequence}}$. Have the students extend the ratio through to all 20 numbers and have them make a conjecture about what happens to the ratio.

Index	Fibonacci Number	Ratios
0	0	
1	1	
2	1	1
3	2	2
4	3	1.5
5	5	1.666667
6	8	1.6
7	13	1.625
8	21	1.615385
9	34	1.619048
10	55	1.617647
11	89	1.618182
12	144	1.617978
13	233	1.618056
14	377	1.618026
15	610	1.618037
16	987	1.618033
17	1597	1.618034
18	2584	1.618034
19	4181	1.618034
20	6765	1.618034

3. Have the students explore the internet for other cultural uses of the Fibonacci Sequence and write a brief description of each.

Assessment

Students will be assessed through observation and peer cooperation, answers expressed on the Exploring Fibonacci worksheet, creation of excel program.

Grading Rubric

Participation and peer cooperation:	30 points
Acceptable responses to worksheet:	30 points
Creation of Excel File using rules:	20 points
Fibonacci internet Exploration:	20 points

Strategies for Differentiation

- The students could explore the arrays on grid paper.
- The excel file could be generated using calculators instead.
- Provide various pictures for students to explore and discuss the culture the picture may be from.
- Have the students explore the graphs of the ratios and then compare the sum of the squares of the ratios and discuss findings.

Exploring Fibonacci Worksheet

Student Exploration Part 1:

Introduction:

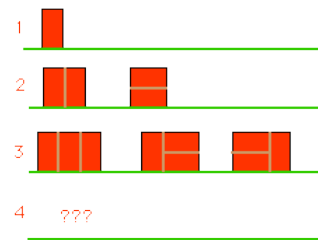
A list of numbers has been given. Find the pattern necessary to complete the remainder of the sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, _____, _____, _____

Explain the pattern used to find the remaining numbers for the sequence.

- Suppose you are a craftsman and you are designing a 2 x 10 rectangle in honor of Mr. Fibonacci. You have been given ten 2 x 1 bricks to cover this rectangle. How many different ways can you lay the bricks. (You will be using domino tiles to represent the bricks.)

Solve the problem by arranging the bricks for a 2 x 1 rectangle first. How many possible ways can they be laid? Next, look at a 2 x 2 construction. How many arrangements are possible? Now try 2 x 3 arrangements. Continue to fill in the chart show to represent the possible ways for the bricks to be laid.



Dimensions	Number of possible arrangements
2 x 1	
2 x 2	
2 x 3	
2 x 4	(answer is not 4)
2 x 5	
2 x 6	
2 x 7	
2 x 8	
2 x 9	
2 x 10	

What pattern emerged within the chart? _____

Exploring Fibonacci Worksheet KEY

Student Exploration Part 1:

Introduction:

A list of numbers has been given. Find the pattern necessary to complete the remainder of the sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610

Explain the pattern used to find the remaining numbers for the sequence.

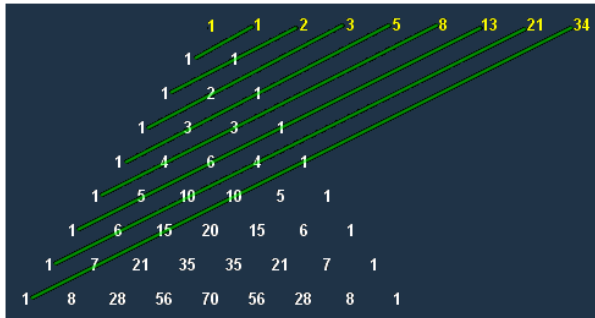
The final term in the sequence is added to the previous term to get the next term. $1+1 = 2$;
 $2+1 = 3$; $3+2 = 5$; etc.

- Suppose you are a craftsman and you are designing a 2×10 rectangle in honor of Mr. Fibonacci. You have been given ten 2×1 bricks to cover this rectangle. How many different ways can you lay the bricks within this rectangle. (You will be using domino tiles to represent the bricks.)

Dimensions	Number of possible arrangements
2 x 1	1
2 x 2	2
2 x 3	3
2 x 4	5
2 x 5	8
2 x 6	13
2 x 7	21
2 x 8	34
2 x 9	55
2 x 10	89

Any particular pattern starting to emerge?
 Fibonacci Sequence

2. Can you identify how the Fibonacci numbers are used in Pascal's Triangle?



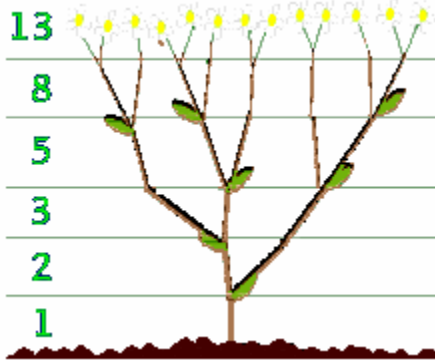
3. Piano keys also take advantage of the famous sequence. Can you explain how?

There are 13 notes in an octave span. From the scale of C to C there are 13 keys: 8 that are white, 5 black keys and they are split into groups of 3 and 2.



4. Tree branching also makes use of the Fibonacci Sequence. Can you identify where?

The number of branches in each section is creating the sequence.



Student Exploration Part 2:

- Using an excel program, create a program that will generate the first 20 Fibonacci sequence of numbers with the first two initial numbers being 0 and 1.

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- Create a third column in the excel program that finds the value of the ratio f_{n+1}/f_n . What fascinating mathematical number appears? **1.618**
- Explore the internet for other cultural uses of the Fibonacci Sequence and write a brief description of each. **Answers will vary.**

The Golden Ratio in Nature

Strand Geometry

Mathematical Objective

Identify how the Golden Ratio produced from the Fibonacci Sequence appears in nature.

Mathematics Performance Expectation(s)

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- Recognize and apply mathematics in contexts outside of mathematics.

Materials/Resource

- Pictures or actual Flowers with various amounts of petals and various pine cones
 - ❖ 3 petals: lily, iris
 - ❖ 5 petals: buttercup, wild rose, larkspur, columbine (aquilegia)
 - ❖ 8 petals: delphiniums
 - ❖ 13 petals: ragwort, corn marigold, cineraria,
 - ❖ 21 petals: aster, black-eyed susan, chicory
- Various Rectangular shapes or pictures of rectangles
- Ruler or tape measure
- Graph paper
- *Exploring the Golden Ratio* worksheet
- You Tube access

Assumption of Prior Knowledge

- Students explored the Fibonacci Sequence and discovered the Golden Ratio of 1.618
- Students are capable of reading a ruler or tape measure accurately.

Introduction: Setting Up the Mathematical Task

This activity is designed for students to explore the Golden Ratio and Fibonacci Sequence and investigate how they apply to many things in the natural world. Each activity will be set up as a station where students investigate versions of the Golden Ratio or find the Fibonacci Sequence.

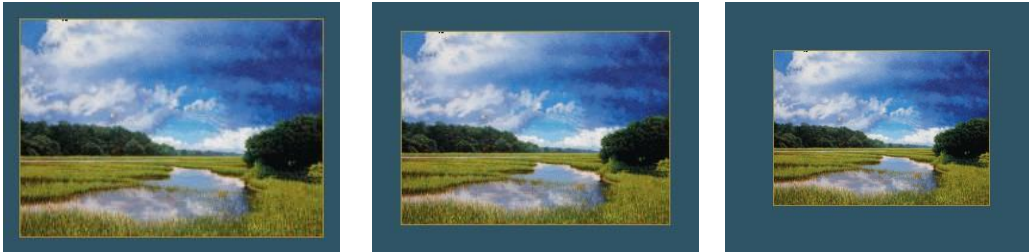
Duration: This project will take approximately one 90 minute class.

Student Exploration 1

Duration: 80 minutes (20 minutes per station)

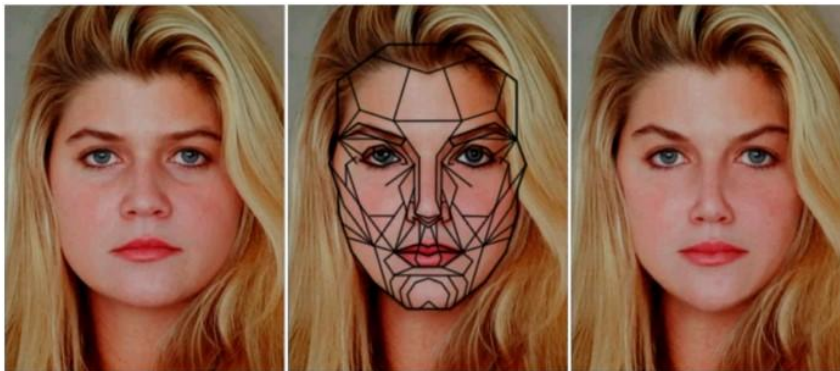
Introduction

Show the following pictures and ask: which of the matted images below do you find most appealing?



If you selected the middle sized border, your sense of aesthetics chose the matte that reflects a golden ratio. The ratio of the area of the matte to the area of the painting is 1.618.

Which picture of this face do you find more appealing, the left or the right?



Original

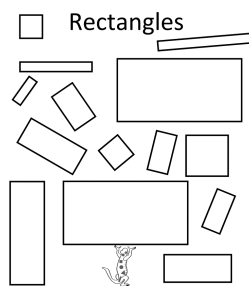
Revise to Marquardt Beauty Mask

Per Beauty Mask

The face on the right has been photo shopped to be perfectly proportional and then shaded to add to its appeal. The human face conforms most closely to Golden Ratio when we smile. You'll be perceived as more beautiful with a warm smile than with a cold-hearted look of anger, arrogance or contempt.

Small Group Work

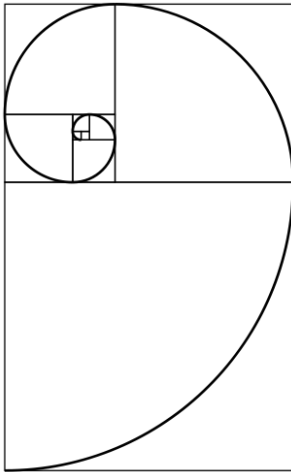
1. Separate the students into groups of 3 or 4 students.
2. Station 1 Rectangles: Provide the students with various rectangular shapes and have them identify the ones they feel are pleasing to the eye. Once the rectangles have been chosen, have the students measure the horizontal length and vertical width of each rectangle and calculate the ratio of length to width. Students should then compare the ratios to the golden ratio and note the similarities. (Alternative for rectangles: have the students bring in rectangular shapes that they feel are eye appealing and then use those for comparison.)



3. Station 2 Flowers and Pine Cones: Have various flower bundles and pine cones available for students to study. (Flowers and pine cones could be brought in by students.) Have them take note of the number of petals per flower and count the number of spirals going clockwise and counter-clockwise in a pine cone. Ask students to make a comparison about the petal numbers and spiral ratio of clockwise : counterclock-wise and make a conjecture.



4. Station 3 Square Spirals: Using grid paper, have the students create the Fibonacci spiral by creating squares whose side measurements correspond to the Fibonacci numbers. First have the students draw a square that measures one square unit. Next have the students draw a second square of one square unit to the left of the square. Third, the students should draw a 2 x 2 square above the ones just drawn. Now, they need to draw a 3 x 3 to the right of the other three squares. The students should continue this pattern until they have filled up the graph paper with similar squares. To create the spiral, the students need to draw an arc starting on the inside of the initial square and have it pass from one corner to the next so that it is continuously passing each new square from corner to corner. The final result should look similar to the given picture.



Ask the students how the lengths of each square relate to the Fibonacci Sequence and where in nature they have seen the spiral that is produced.

5. Station 4: The Human Golden Ratio

Have the students find the measurements of various facial parts and then calculate the ratio between facial features.

Body Part	Measurement
Top of head to chin	
Width of head	
Top of head to pupil	
Pupil to lip	
Tip of nose to chin	
Lips to chin	
Length of lips	
Width of nose	

	Ratio
Top of your head to your chin / width of your head	
Top of your head to your pupil / your pupil to your lip	
Tip of your nose to your chin/ your lips to your chin	
Tip of your nose to your chin/ your pupil to the tip of your nose	
Length of your lips/ the width of your nose.	

Whole Class Sharing/Discussion (10 minutes)

- Discuss findings of each station and relate it back to the introduction about how the Golden Ratio is used in our everyday lives.
- Ask students to create an exit ticket that explains what the golden ratio is and how it relates to them personally.

Student/Teacher Actions:

- Monitor student participation at each station
- Check for understanding at each station, especially the spiral squares and pine cone stations
- Students should be measuring and calculating at each station

Assessment

Students will be graded based on responses to observations at each station

Grading rubric

Participation at each station	10 points per station (40 points)
Responses per station	10 points per station (40 points)
Exit Ticket	20 points

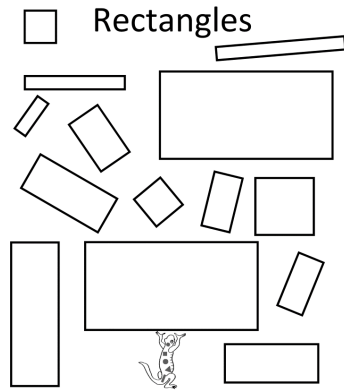
Strategies for Differentiation

- Have squares for the spiral square cut out to length and have the students reconstruct the spiral.
- Have the students research ways to produce the Golden Ratio and have them do a presentation on their research.
- Have students measure various rectangles found within the room.
- Have students read the Elliott Wave Principle and explore how it relates today's issues.

Exploring the Golden Ratio Worksheet

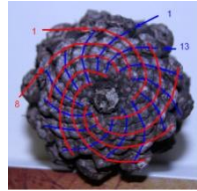
1. Station 1: Rectangles

Using the various rectangles provided, identify the ones that you feel are appealing to the eye. Measure the horizontal length and vertical width of the rectangles you chose. Calculate the ratio of the length to the width and share with your group members. Once all members have calculated the ratios, compare the results to the Golden Ratio.



2. Station 2: Flowers and Pine Cones

a) Using the bundle of flowers provided, create a chart that indicates the type of flower you observed and the number of petals on the flower.



b) Looking at a pine cone from the top view, count the number of spirals going clockwise and counter-clockwise. Find the ratio of clockwise spirals to counter-clockwise spirals for each pine cone.

c) Note any observations you make regarding the flower petals and the ratios of the pine cones.

3. Station 3: Square Spirals

- a. Using the graph paper, you will now create a spiral by creating squares whose side measurements are always the new measurement of the next square.
 1. First draw a square that measures one square unit.
 2. Next draw a second square of one square unit to the left of the square.
 3. Third, draw a 2 x 2 square above the squares just drawn; making sure that one side of your square is the length of the two squares just drawn.
 4. Now, draw a 3 x 3 to the right of the other three squares.
 5. Continue this pattern until you have filled up the graph paper with similar squares.
- b. To create the spiral, you need to draw an arc starting on the inside of the initial square and have it pass from one corner to the next so that it is continuously passing each new square from corner to corner. Check with the teacher to verify you are drawing the spiral correctly.
- c. List the side measurements of each square. What do you notice?

- d. Where in nature have you seen a spiral similar to the one generated by the squares?

4. Station 4: Human Golden Ratio

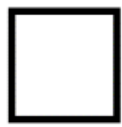
a. Find the following measurements

Body Part	Measurement
Top of head to chin	
Width of head	
Top of head to pupil	
Pupil to lip	
Tip of nose to chin	
Lips to chin	
Length of lips	
Width of nose	

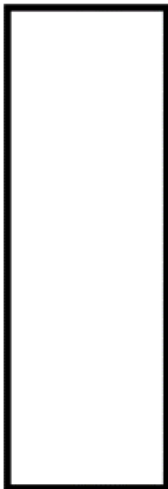
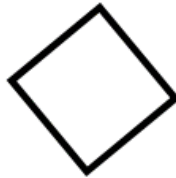
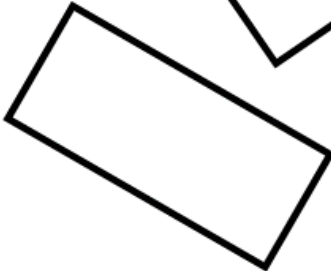
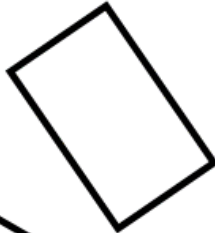
b. Calculate the following ratios

	Ratio
Top of your head to your chin / width of your head	
Top of your head to your pupil / your pupil to your lip	
Tip of your nose to your chin/ your lips to your chin	
Tip of your nose to your chin/ your pupil to the tip of your nose	
Length of your lips/ the width of your nose.	

c. How do your ratios compare to the Golden Ratio?



Rectangles



Flowers



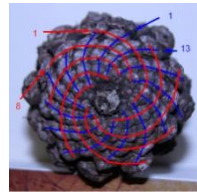
Exploring the Golden Ratio Worksheet (KEY)

1. Station 1: Rectangles

Using the various rectangles provided, identify the ones that you feel are appealing to the eye. Measure the horizontal length and vertical width of your rectangle. Calculate the ratio of the length to the width and share with your group members. Once all members have calculated the ratios, compare the results to the Golden Ratio. **The eye pleasing rectangles should produce a ratio close to 1.6.**

2. Station 2: Flowers and Pine Cones

- a) Using the bundle of flowers provided, create a chart that indicates the type of flower you observed and the number of petals on the flower.



- b) Looking at a pine cone from the top view, count the number of spirals going clockwise and counter-clockwise. Find the ratio of clockwise spirals to counter-clockwise spirals for each pine cone.
- c) Note any observations you make regarding the flower petals and the ratios of the pine cones. **The flower petals are the numbers from the Fibonacci Sequence while the pine cone ratio should be the Golden Ratio**

3. Station 3: Square Spirals

- a. Using the graph paper, you will now create a spiral by creating squares whose side measurements are always the new measurement of the next square.
- 1) First draw a square that measures one square unit.
 - 2) Next draw a second square of one square unit to the left of the square.
 - 3) Third, draw a 2 x 2 square above the squares just drawn; making sure that one side of your square is the length of the two squares just drawn.
 - 4) Now, draw a 3 x 3 to the right of the other three squares.
 - 5) Continue this pattern until you have filled up the graph paper with similar squares.
- b. To create the spiral, you need to draw an arc starting on the inside of the initial square and have it pass from one corner to the next so that it is

continuously passing each new square from corner to corner. Check with the teacher to verify you are drawing the spiral correctly.

- c. List the side measurements of each square. What do you notice?
1, 1, 2, 3, 5, 8, etc which is the Fibonacci Sequence
- d. Where in nature have you seen a spiral similar to the one generated by the squares? **Sample answers: Sea Shells, pine cones**

4. Station 4: Human Golden Ratio

How do your ratios compare to the Golden Ratio?

The ratios should be close to 1.6

Fibonacci Gauge

Strand Geometry

Mathematical Objective

The student will create a Fibonacci gauge in order to create and measure various objects.

Mathematics Performance Expectation(s)

MPE. 3. The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation.

MPE. 7. The student will use similar geometric objects in two- or three-dimensions to solve real-world problems about similar geometric objects.

Related SOL

G.14 The student will use similar geometric objects in two- or three-dimensions to compare ratios between side lengths, perimeters, areas, and volumes and solve real-world problems about similar geometric objects.

NCTM Standards

- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
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- solve problems involving scale factors, using ratio and proportion;
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- Recognize and apply mathematics in contexts outside of mathematics.

Materials/Resources

- Card stock paper and fasteners
- Scissors
- Rulers
- Fibonacci Gauge Template
- Various items to measure
- You Tube access

Assumption of Prior Knowledge

- The student should be able to accurately measure objects using millimeters
- The student should be familiar with the Golden Ratio

Introduction: Setting Up the Mathematical Task

As an introduction to the Fibonacci Gauge, have the students watch the first two minutes of the you tube video “Fibonacci Gauge”:

<http://www.youtube.com/watch?v=5Xgw84Kwrh8> and the video clip “Golden Ratio Caliper Examples” <http://www.youtube.com/watch?v=hWzbusvWGpE> After the introduction, provide each student with the necessary tools to create the gauge and model how to make the tool. Once assembled, they will use the tool to measure various items found within the class room.

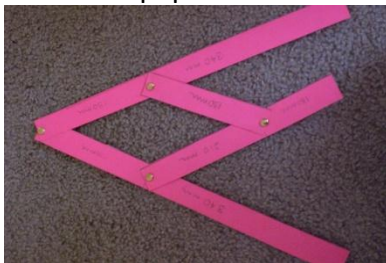
Duration: This project will take approximately one 90 minute class.

Student Exploration 1

Duration: 30 minutes

Individual Work

1. Provide each student 4 fasteners, a scissor and ruler, the gauge template and a piece of card stock paper.



2. Instruct the students to cut out 2 long pieces that measure 470mm, a medium size piece that measures 390mm and a short piece that measures 130mm. Each piece should be about the same width (approximately $\frac{1}{2}$ inch).
3. On the two long pieces, the students need to mark a line 130mm from one end to know where to place one set of fasteners. The middle piece needs a mark at 210 mm for a fastener.

4. Students are to punch a hole in the center of the given marks as well as a hole at beginning end of the 130mm measurement for the long pieces. Students need to punch a hole at both ends of the small 130mm piece and a hole at the beginning of the 210mm piece
5. Have students assemble the gauge according to the given picture.
6. Instruct students to find five things within the room that demonstrate the Golden Ratio based on the Fibonacci Gauge created. (Examples students may not consider: Ipod, ID card, television, paper)

Student Exploration 2

Duration: 60 minutes

Individual Work

1. Continue the video on using the Fibonacci Gauge .
2. Discuss with students other possible uses of the gauge besides furniture making
3. Using anything learned about the Fibonacci Sequence, Golden Ratio, or Fibonacci Gauge, allow students to design their own picture by making use of the gauge.

Student/Teacher Actions

- Monitor student actions and assist where necessary.
- Students should be locating and measuring possible objects within the room.
- Giving students ideas for possible creations that may be made with the gauge.

Assessment

Students will be graded based on participation of creating the gauge and making use of its ability to measure objects with a ratio of 1 : 1.618

Grading rubric

Participation in discussion	10 points
Accurate gauge creation	45 points
5 points per accurate length pieces	
5 points per accurate positioning of fasteners	
5 points for functioning gauge	
Finding objects of a golden ratio	10 points (2 per object-5 minimum)
Golden Ratio drawing	35 points
5 points neatness	
5 points for color	
5 points for uniqueness	
15 points accuracy	

Strategies for Differentiation

- Have the pieces cut and marked so only assembly is required
- Allow students to create gauge from own materials at home or encourage students to bring an example of one that parents have.
- Have students research and create their own video clip that shows people making use of the Fibonacci Gauge.

References

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